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Monte Carlo Generators and the CCFM Equation[a]

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We discuss three implementations of the CCFM evolution equations in event generator programs. We find that some of them are able to describe observables such as forward jet rates in DIS at HERA, but only if the so-called consistency constraint is removed. We also find that these results are sensitive to the treatment of non-singular terms in the gluon splitting function.

1 Introduction

Small-x final states are central to the understanding of small-x evolution in general. To understand final-state properties such as forward jet rates and transverse energy flow, it is important to have event generators which give a good description of the experimental data. It has, however, turned out to be difficult to produce such an event generator.

To leading double-log accuracy, the CCFM [1] evolution should be the best way of describing small-x final states, and some attempts to implement CCFM into an event generator has been made e.g. SMALLX [2] and LDCMC [3]. Below we will discuss some recent developments of these generators as well as the new CASCADE [4] generator for CCFM.

2 CCFM and the SMALLX and CASCADE programs

The implementation of the CCFM [1] parton evolution in the forward evolution Monte Carlo program SMALLX is described in detail in [2], and we have already reported on some recent developments in [4] and [5]. The main new ingredient is a modification in the treatment of the non-Sudakov form factor, Δ_{ns}, which allows for the removal of the so-called consistency constraint. Rather than using the simple form, $\log \Delta_{ns} = -\bar{\alpha}_s \log(1/z_i) \log(k_{ti}^2/z_i q_{ti}^2)$, which requires

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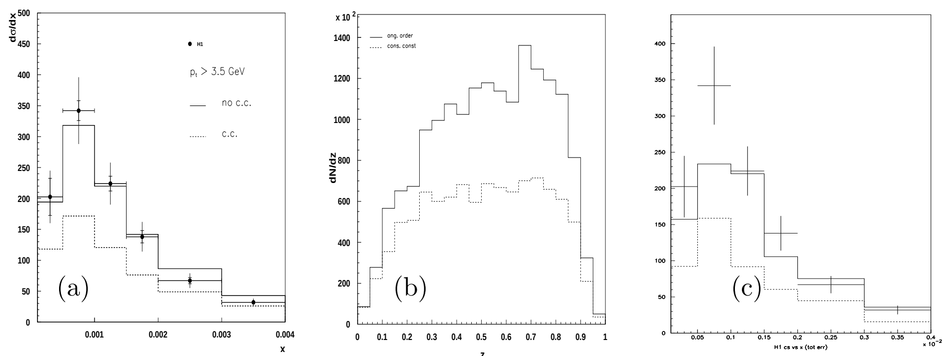

Figure 1: (a) The cross section for forward jet production as a function of x, compared to H1 data [7]. (b) The values of z in the initial state cascade for events satisfying the selection of forward jet production. The solid (dashed) lines are the prediction of the Monte Carlo without (with) applying the "consistency constraint" (c.c.). (c) is the same as (a) but using the LDCMC program including only gluonic ladders with (dotted line) and without (full line) the non-singular terms in the gluon splitting function.

the consistency constraint $k_{ti}^2 > z_i q_{ti}^2$ in order to be below unity, the full form [6] is used:

$$\log \Delta_{ns} = -\bar{\alpha}_s \log\left(\frac{z_0}{z_i}\right) \log\left(\frac{k_{ti}^2}{z_0 z_i q_{ti}^2}\right), z_0 = \begin{cases} 1 & \text{if } k_{ti}/q_{ti} > 1 \\ k_{ti}/q_{ti} & \text{if } z < k_{ti}/q_{ti} \leq 1 \\ z & \text{if } k_{ti}/q_{ti} \leq z \end{cases} \quad (1)$$

This form factor is well behaved for all emissions and gives no suppression in the region $k_{ti}/q_{ti} \leq z$ where then $\Delta_{ns} = 1$.

In Fig. 1a the prediction for the forward jets is shown. The data are nicely described. Including the consistency constraint, the x dependence of the total cross section changes, but a similarly good description of F_2 is obtained by changing the infrared cutoff, Q_0. However the forward jet cross section is reduced.

The effect of the consistency constraint is well illustrated in the spectrum of the values of the splitting variable z for events that satisfy the criteria of forward jet production (Fig. 1b). Only about half of the events satisfy the consistency constraint, and especially medium and large values of z are rejected. Furthermore we notice that in general z is not very small thus showing that even in forward jet production we are far away from the asymptotic region, where the small x approximation is valid.

The CASCADE program which implements CCFM evolution in a backward evolution algorithm, gives results which agree well with the ones from SMALLX. It should also be noted that these programs also are able to reproduce other data, such as F_2^c and photo-production of J/Ψ as measured at HERA.

3 The Linked Dipole Chain Model and the LDCMC program

The Linked Dipole Chain Model [8], LDC, is a reformulation of the CCFM evolution. The main idea is to reinterpret the non-Sudakov form factor as a normal Sudakov for the no-emission probability in some phase-space region. By redefining the division between initial- and final-state splittings (which is done by angular ordering in CCFM), requiring the transverse momentum of a gluon emitted in the initial-state to be larger than the minimum transverse momenta of the connecting propagators, $q_{i\perp}^2 > \min(k_{i\perp}^2, k_{i-1\perp}^2)$, an extra weight is given to each splitting corresponding to the sum of all emissions which in this way are treated as final-state. In the limit of emissions which are strongly ordered both in longitudinal and transverse momenta of the propagating gluon, it can then be shown that if the kinematical constraint is applied, this sum exactly cancels the non-Sudakov. It has, however, not been proven that this holds if the consistency constraint is relaxed.

LDCMC is able to give a good description of F_2, but underestimates the forward jet rates by approximately a factor 2. Much effort has lately been put into the understanding of the differences between SMALLX and CASCADE on one hand and LDCMC on the other. Some of this work was reported on in [5]. One difference between LDCMC and SMALLX is that the latter only includes the singular parts of the gluon splitting function, i.e. the $1/z$ and the $1/(1-z)$ terms, where the $1/z$ term is regularized by the non-Sudakov:

$$\tilde{P}_g(z) = \Delta_{ns}\frac{1}{z} + \frac{1}{1-z}. \tag{2}$$

LDCMC, however, uses the full splitting function including the non-singular terms (where the non-Sudakov has been canceled):

$$P_g(z) = \frac{1}{z} + \frac{1}{1-z} - 2 + z(1-z). \tag{3}$$

In the limits of z close to zero or one, which is the limit in which the CCFM equation is derived, the non-singular terms should be negligible. But, as seen in fig. 1b, the typical z-values in events with forward jets are around 0.5 where the non-singular terms may reduce the splitting function with almost a factor 2. It is intriguing to note that the forward jet rates obtained by LDCMC is approximately a factor two below those obtained with SMALLX (which agrees with data). Indeed, preliminary investigations show that removing the non-singular terms from LDCMC does increase the forward jet rates as seen in fig. 1c although not with as much as a factor of two [9].

4 Open questions

The fact that the results from the SMALLX, CASCADE and LDCMC programs depends strongly on the treatment of the consistency constraint and on the non-singular terms in the gluon splitting functions, means that also the CCFM equation as such is sensitive to these non-leading effects for observables such as the forward jet rates.

To include the non-singular terms in the gluon splitting function in the CCFM equation is not completely straight forward. Simply adding them to eq. 2 may result in negative splitting probabilities. But in the limits of z close to zero or one, it is allowed to let the non-Sudakov in eq. 2 multiply also the $1/(1-z)$ term: $\tilde{P}_g = \Delta_{ns} P_g^{sing}(z) = \Delta_{ns}(1/z + 1/(1-z))$, in which case it is possible to replace the $P_g^{sing}(z)$ with the full splitting function of eq. 3. Preliminary investigations [9] show that doing this in the SMALLX program indeed gives a large effect, although more studies are needed to quantify the influence on the results for e.g. forward jet rates.

In conclusion, it is still an open question whether the CCFM evolution equation is an appropriate way of describing small-x final states or not. Much more work is needed, both phenomenological work using the event generators and purely theoretical studies of the CCFM evolution, to understand how to correctly treat the non-leading effects which seem to be important in the description of data.

References

1. M. Ciafaloni, *Nucl. Phys.* **B 296** (1988) 49,
   S. Catani, F. Fiorani, G. Marchesini, *Phys. Lett.* **B 234**(1990)339; *Nucl. Phys.* **B 336**(1990)18,
   G. Marchesini, *Nucl. Phys.* **B 445** (1995) 49.
2. G. Marchesini, B. Webber, *Nucl. Phys.* **B 349** (1991) 617,
   G. Marchesini, B. Webber, *Nucl. Phys.* **B 386** (1992) 215.
3. H. Kharraziha, L. Lönnblad, *JHEP* **98003:006** (1998).
4. H. Jung, 'CCFM prediction on forward jets and $F_2\ldots$', in *Proc. of the Workshop on Monte Carlo Generators*, edited by A. Doyle, G. Grindhammer, G. Ingelman, H. Jung (DESY, Hamburg, 1999).
5. H. Jung, L. Lönnblad, *J. Phys. G: Nucl. Part. Phys.* **26** (2000) 707.
6. J. Kwiecinski, A. Martin, P. Sutton, *Phys. Rev.* **D 52** (1995) 1445.
7. H1 Collaboration, C. Adloff et al., *Nucl. Phys.* **B 538** (1999) 3.
8. B. Andersson, et al., *Nucl. Phys.* **B 467** (1996) 443,
   B. Andersson, et al., *Z. Phys.* **C 71** (1996) 613.
9. H. Jung, L. Lönnblad, Work in progress.